# BOUNDARY LAYER OF A MICROSTRUCTURAL FLUID ON A PLATE

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A numerical solution of the self-similar problem of the boundary layer of a micropolar model fluid on a semiinfinite plate is obtained.

The ideas of the Prandtl boundary layer, which have been widely developed in the classical hydromechanics of a viscous fluid [1], have been successfully extended to rheologically complex media. A systematic account and analysis of the results obtained for non-Newtonian fluids are given in [2], for example. It is of interest to apply the hypothesis of a boundary layer in asymmetric fluid mechanics [3, 4], which provides a viewpoint for description of the behavior of such structurally inhomogeneous media as liquid crystals, ferrofluids, suspensions, blood, etc.

In [5] four different types of equations of a plane boundary layer for a micropolar fluid model [6] were derived for the first time. The question of the existence of self-similar problems for a two-dimensional micropolar boundary layer was analyzed in [6] with the aid of the group method [7].

In the present work we obtain a numerical solution of the self-similar problem of the boundary layer of a micropolar fluid of the fourth type on a semi-infinite thin plate (the generalized Blasius problem). The question of the effect of rheological coefficients and different kinds of boundary conditions for microrotations on the flow regime is investigated.

1. We consider a steady flow of an incompressible micropolar fluid along a plane thin semi-infinite plate. The origin of the Cartesian coordinate system is taken at the front point of the plate. The x axis is directed along the plate and the y axis along a normal to it. Let $U_\infty$ be the velocity of the incident flow, which is parallel to the y axis. The equations of a micropolar boundary layer of the fourth type then take the following form [5]:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu(1+s)\frac{\partial^2 u}{\partial y^2}+\nu s\frac{\partial w}{\partial y},$$

$$u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}=\nu m\frac{\partial^2 w}{\partial y^2},\quad \frac{\partial u}{\partial x}+\frac{\partial w}{\partial y}=0. \tag{1.1}$$

The boundary conditions are assigned in the form

$$u(x,\ 0)=v(x,\ 0)=0,\quad w(x,\ 0)=w_0, \tag{1.2}$$

where $w_0$ is the still undetermined microrotation on the solid wall,

$$u(x,\ y)\to U_\infty,\quad w(x,\ y)\to 0,\quad y\to\infty. \tag{1.3}$$

Following classical boundary-layer theory [1], we introduce a stream function $\Psi(x, y)$, such that the continuity condition is satisfied

$$u=\frac{\partial\Psi}{\partial y},\quad v=-\frac{\partial\Psi}{\partial x}.$$

Then, in the usual way, we replace $\Psi(x, y)$ by the dimensionless stream function $f(\eta)$:

$$\Psi(x,\ y)=\sqrt{2\nu xU_\infty}\,f(\eta),\quad \eta=y\sqrt{U_\infty/2\nu x}. \tag{1.4}$$

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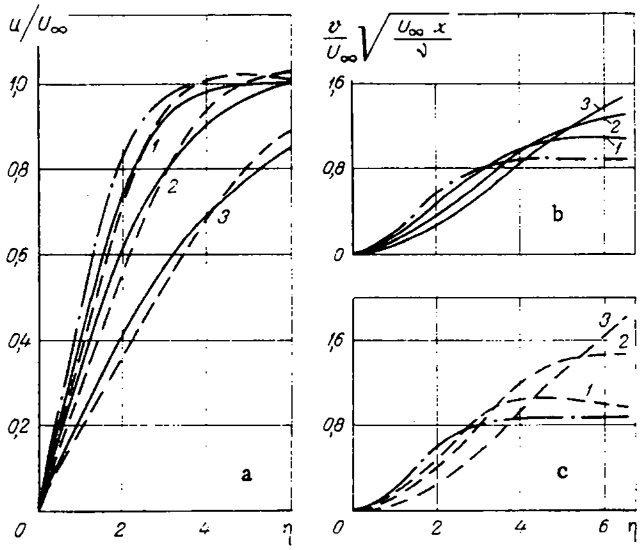


Fig. 1. Distribution of longitudinal and transverse velocity components for different values of rheological parameters: 1) s = 0.5; 2) 2.5; 3) 10; the solid lines correspond to m = 0.5, the dashed lines to m = 10, and the dot-dash lines to a Newtonian fluid.

In the new variables the velocity vector components have the form

$$u = U_\infty f', \quad v = \sqrt{\frac{\nu U_\infty}{2x}}(\eta f' - f).$$

In place of the microrotation component w(x, y) we introduce the dimensionless microrotation function $\varphi(\eta)$:

$$w(x, y) = U_\infty \sqrt{\frac{U_\infty}{2\nu x}}\,\varphi(\eta). \tag{1.5}$$

When (1.4) and (1.5) are taken into account, the initial system of equations (1.1) reduces to a system of ordinary differential equations

$$(1 + s) f''' + ff'' + s\varphi' = 0, \quad m\varphi'' + (f\varphi)' = 0. \tag{1.6}$$

We replace the boundary conditions (1.2) and (1.3) by the conditions

$$f(0) = f'(0) = 0, \quad \varphi(0) = \frac{4w_0}{U_\infty}\sqrt{\frac{\nu x}{U_\infty}}, \tag{1.7}$$

$$f'(\eta) \to 1, \quad \varphi(\eta) \to 0, \quad \eta \to \infty. \tag{1.8}$$

2. The most common boundary condition on a solid wall for microrotation is the no-slip condition [8], i.e, the condition $w_0 = 0$. As indicated in [5], however, in this case for equations of the fourth type of micropolar boundary layer $w \equiv 0$ throughout the flow.

It was suggested in [9] that the condition of zero value of the antisymmetric part of the stress tensor

$$\frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\Big|_{y=0} = w_0 \tag{2.1}$$

on the wall should be used as a boundary condition for microrotation on a solid wall.

In this case the second boundary condition (1.7) takes the form

$$\varphi(0) = -\frac{1}{2} f''(0). \tag{2.2}$$

TABLE 1. Boundary-Layer Thicknesses and Friction Stress on Plate in Relation to Values of Rheological Parameters of Fluid

| $s$ | $\delta/\sqrt{2}$ | $\delta^*\sqrt{\frac{U_\infty}{2\nu x}}$ | $\delta^{**}\sqrt{\frac{U_\infty}{2\nu x}}$ | $\tau_0\sqrt{\frac{2x}{\rho\mu U_\infty^3}}$ |
|---|---|---|---|---|
| | | $m = 0,5$ | | |
| 0 | 3,5 | 1,72 | 0,664 | 0,332 |
| 0,1 | 3,6 | 1,76 | 0,681 | 0,340 |
| 0,5 | 4,0 | 1,92 | 0,747 | 0,373 |
| 1,0 | 4,4 | 2,10 | 0,827 | 0,413 |
| 2,5 | 5,8 | 2,58 | 1,046 | 0,523 |
| 5,0 | 7,6 | 3,24 | 1,359 | 0,680 |
| 10,0 | 10,1 | 4,30 | 1,877 | 0,935 |
| | | $m = 10$ | | |
| 0,1 | 3,6 | 1,767 | 0,673 | 0,336 |
| 0,5 | 5,7 | 1,941 | 0,713 | 0,356 |
| 1,0 | 6,2 | 2,136 | 0,768 | 0,383 |
| 2,5 | 7,5 | 2,631 | 0,939 | 0,465 |
| 5,0 | 9,4 | 3,262 | 1,153 | 0,577 |
| 10,0 | 10,2 | 4,329 | 1,560 | 0,780 |

TABLE 2. Boundary-Layer Thicknesses and Friction Stress on Plate in Relation to Boundary Parameter

| $\alpha$ | $\delta/\sqrt{2}$ | $\delta^*\sqrt{\frac{U_\infty}{2\nu x}}$ | $\delta^{**}\sqrt{\frac{U_\infty}{2\nu x}}$ | $\tau_0\sqrt{\frac{2x}{\rho\mu U_\infty^3}}$ |
|---|---|---|---|---|
| —2,5 | 1,4 | 0,604 | 0,009 | 0,004 |
| —1,5 | 1,9 | 1,076 | 0,327 | 0,163 |
| —0,5 | 3,4 | 1,689 | 0,654 | 0,327 |
| —0,2112 | 4,0 | 1,920 | 0,747 | 0,373 |
| 0,1 | 4,4 | 2,200 | 0,845 | 0,422 |
| 0,75 | 5,2 | 2,974 | 1,042 | 0,521 |
| 1,25 | 6,0 | 3,399 | 1,182 | 0,589 |
| 1,75 | 8,2 | 7,034 | 1,288 | 0,644 |

With boundary condition (2.2) the considered problem becomes self-similar. The boundary-value problem (1.6)-(1.8) with (2.2) taken into account was reduced in the present work to a Cauchy problem with certain, initially arbitrary, values of $A = f''(0)$ and $B = \varphi'(0)$. We integrated the obtained Cauchy problem by the Runge–Kutta method, using "ranging" [10] to select values of A and B at which the boundary conditions (1.8) are satisfied. The problem was solved for different values of the parameters s and m on a BÉSM-4 computer.

The distribution of the longitudinal and transverse velocity components is shown in Fig. 1. For any values of s and m the velocity profiles become less filled than the Blasius profile (dot–dash line). Figure 1b,c indicates that the transverse velocity component increases more slowly at first, but subsequently attains higher values on the outer boundary of the boundary layer than in the case of a Newtonian fluid. For $m > 1$ within the boundary layer the longitudinal velocity component attains higher values than the velocity of the external flow, which is indicated by the characteristic maxima on Fig. 1a,c for $m = 10$.

The obtained solutions for the velocity distribution and microrotation allow us to calculate the stresses and the couple stresses in the boundary layer

$$t_{xy} = U_\infty \sqrt{\frac{\rho\mu U_\infty}{2x}}\,(f'' - s\varphi), \quad t_{yx} = U_\infty \sqrt{\frac{\rho\mu U_\infty}{2x}}\,[(1 + s)f'' + s\varphi],$$

$$m_{xz} = -\frac{\nu_2 U_\infty}{2\rho x}\sqrt{\frac{U_\infty}{2\nu x}}\,(\eta\varphi'' + \varphi), \quad m_{yz} = \frac{\nu_2 U_\infty^2}{2\rho\nu x}\varphi'.$$

The local tangential stress on the plate is

$$\tau_0(x) = U_\infty \sqrt{\frac{\rho\mu U_\infty}{2x}}\,[(1+s)f''(0)+s\varphi(0)].$$

The displacement thickness and the momentum thickness are given by the relations [1]

$$\delta^* = \int_0^\infty \left(1-\frac{u}{U_\infty}\right)dy = \sqrt{\frac{2\nu x}{U_\infty}}\,[\eta_1 - f(\eta_1)],$$

$$\delta^{**} = \int_0^\infty \frac{u}{U_\infty}\left(1-\frac{u}{U_\infty}\right)dy,$$

where $\eta_1$ corresponds to a point lying outside the boundary layer.

Table 1 gives the main flow characteristics in relation to the change in the dimensionless parameters s and m. Table 1 shows that the boundary-layer thicknesses and friction stress on the plate increase with increase in s. This is qualitatively consistent with the results of calculations for the boundary layer of suspensions of solid particles in water [11]. With increase in m and fixed s the relative boundary layer thickness $\delta$ and the displacement thickness $\delta^*$ increase, while the momentum thickness $\delta^{**}$ and friction stress $\tau_0$ on the plate decrease.

3. It is apparent from the second boundary condition (1.7) that if we assign on the plate the condition of variation of the rate of microrotation in accordance with the law

$$w_0 = \frac{\alpha U_\infty}{4}\sqrt{\frac{U_\infty}{\nu x}}, \tag{3.1}$$

where $\alpha$ is a constant that depends on the physical characteristics of the surface in the flow and is a measure of the interaction of the fluid and solid wall, the considered problem also becomes self-similar.

The velocity distribution obtained for s = 0.5 and m = 1 and different values of $\alpha$ is shown in Fig. 2a, b. A characteristic feature of flows with boundary condition (3.1) is the appearance of S-shaped profiles of the longitudinal velocity component (curves 6 and 7, Fig. 2a) and "induced flows" near the plate (1-3, Fig. 2). Calculations showed that the distortion of the S-shaped velocity profiles increases with increase in m. The transverse velocity component increases more rapidly with increase in the negative value of $\alpha$ at small $\eta$ than in the case of Newtonian fluids (dot-dash lines), but at the outer boundary of the boundary layer its value is reduced, as Fig. 2b clearly shows.

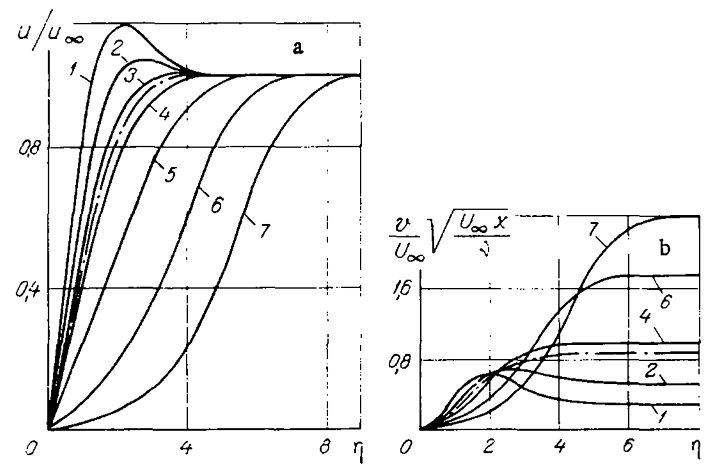


Fig. 2. Distribution of: a) longitudinal velocity component; b) transverse velocity component for s = 0.5, m = 1. 1) $\alpha$ = −2.5; 2) −1.5; 3) −0.5; 4) −0.5f''(0) = −0.2112; 5) 0.75; 6) 1.25; 7) 1.75.

The values of the relative boundary-layer thickness, displacement thickness, momentum thickness, and local tangential stress on the plate in relation to $\alpha$ are given in Table 2.

It should be noted that the obtained solutions include, as a special case of the considered problem, the solution for the boundary layer of a Newtonian fluid. For this we must put $s = 0$.

## NOTATION

x, y, z, Cartesian coordinates; u, v, velocity vector components; w, microrotation vector component; $\nu$, $\nu_1$, $\nu_2$, coefficients of shear, rotational, and couple viscosity; $\mu = \nu/\rho$, dynamic coefficient of shear viscosity; $\rho$, density; $w_0$, microtation on plate; $U_\infty$, velocity of incident flow; $s = \nu_1/\nu$, $m = j\nu_2/\nu$, dimensionless rheological parameters of fluid model; j, microinertia; A, B, $\alpha$, constants; $\Psi$, stream function; $\eta$, self-similar variable; f, $\varphi$, dimensionless stream and microrotation functions; $t_{xy}$, $t_{yx}$, stress tensor components; $m_{xy}$, $m_{yx}$, couple stress tensor components; $\tau_0$, local tangential stress on plate; $\delta$, relative boundary-layer thickness; $\delta^*$, displacement thickness; $\delta^{**}$, momentum thickness.

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# HEAT TRANSFER IN A STRETCHED BICOMPONENT FILM

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The temperature distribution in a bicomponent film during nonisothermal stretching is discussed without allowance for dissipative heating.

Bicomponent films (laminates) are increasingly used, because such films are of particularly good performance, e.g., in the production of crimped materials, where the crimping arises from differences in elasticity of the components. A polymer characteristically has very marked temperature dependence of the elastic parameters, so the working temperature is a major parameter in production of crimped bicomponent material. Appropriate thermal conditions can be provided in pulling such films in order to control the crimping [1, 2].

Here we consider the temperature pattern in such a film during nonisothermal stretching.

Figure 1 shows the drawing scheme.

We use an immobile Euler coordinate system with the x axis coincident with the direction of motion of the film, while the y axis is perpendicular to that direction, and the origin and the x axis are equidistant from the outer surfaces of the film.

We assume a linear distribution for the axial velocity of the film in the drawing zone [3-5]:

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